

PROOF101, Spring 2026, Slides 02 (Appendix)

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Some Taxonomy of Proof Systems:

Implicational, Minimal, Intuitionistic, Classical

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Classifying Proof Systems of Propositional/First-Order Logic

Based on the logical connectives that the proof system processes:

- Every proof system includes rules for the *introduction* and *elimination* of the connective ' \rightarrow '; in our notation, these are $(\rightarrow I)$ and $(\rightarrow E)$.
- If $(\rightarrow I)$ and $(\rightarrow E)$ are the only rules, we say this is the **implicational version** of propositional logic. *
- If in addition to $(\rightarrow I)$ and $(\rightarrow E)$ the system includes a rule for the elimination of absurdity ' \perp ', we say this is the **implicational version with $(\perp E)$** . †
- If in addition to $(\rightarrow I)$ and $(\rightarrow E)$ the system includes rules for the *introduction* and *elimination* of ' \wedge ' and ' \vee ', i.e., the rules $\{(\wedge I), (\wedge E), (\vee I), (\vee E)\}$, we say this is the **minimal version** of propositional logic.
- If in addition to the rules of the *minimal version* the system includes the rule for eliminating absurdity ' \perp ', i.e., $\{(\rightarrow I), (\rightarrow E), (\wedge I), (\wedge E), (\vee I), (\vee E), (\perp E)\}$, we get the **minimal version with $(\perp E)$** , which is better known as the **intuitionistic version** of propositional logic.

* If you care to read more, go to the Wikipedia page: [Implicational Propositional Logic](#) .

† $(\perp E)$ is also known as [Principle of Explosion](#) and sometimes invoked by a Latin name *ex falso quodlibet* or just *ex falso*.

Classifying Proof Systems of Propositional/First-Order Logic

- Finally, if we add to the *intuitionistic version* any one of the four following rules:
 - **(PBC)**, also known as *reductio ad absurdum*,
 - **(LEM)**, also known as *tertium non datur*,
 - **($\neg\neg$ E)**,
 - **(Peirce's)**,

we say this is the **classical version** of propositional logic.

FACT: The four preceding rules are inter-derivable using only **(\rightarrow I)** and **(\rightarrow E)**.

EXERCISE: Show that **(\perp E)** is derivable from **(PBC)** (or from any of the three other preceding rules).

Hence, another way of obtained *classical propositional logic* is to augment *minimal propositional logic* with any of the four preceding rules.

- The preceding classification into *implicational*, *implicational with (\perp E)*, *minimal*, *intuitionistic*, and *classical*, applies again when we later extend *propositional logic* to *first-order logic*.

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